Section 3.3: Matrix Operations

Addition and Subtraction of Matrices

If A and B are two matrices of the same size,

1. A + B is the matrix obtained by adding the corresponding entries in the two matrices.

2. A – B is the matrix obtained by subtracting the corresponding entries in B from A.

Laws for Matrix Addition

If A, B, and C are matrices of the same dimension, then

1. A + B = B + A

2. (A + B) + C = A + (B + C)

Example 1: Refer to the following matrices: If possible,

$$A = \begin{bmatrix} 8 & -3 & 1 \\ 0 & -9 & -4 \\ 9 & 6 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} -5 & 4 & -1 \\ 8 & 4 & 8 \\ 10 & 15 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 10 & -8 & 3 \\ 5 & -4 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & 1 & 3 \\ 8 & 5 & 1 \end{bmatrix}$$

a. compute $A - B$

b. compute B + C.

c. compute D + C.

Scalar Multiplication

A **scalar** is a real number.

Scalar multiplication is the product of a scalar and a matrix. To perform scalar multiplication, each element in the matrix is multiplied by the scalar; hence, it "scales" the elements in the matrix

Example 2: Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} -1 & 4 \\ -7 & 9 \end{pmatrix},$ and $C = \begin{pmatrix} 1 & 2 & 3 \\ -6 & -9 & 1 \end{pmatrix}$ find, if possible,

a. -3C

b. -2B – A

c. 3B + 2C

Transpose of a Matrix

If A is an m x n matrix with elements a_{ij}, then the **transpose** of A is the n x m matrix A^T with elements a_{ji}.

$$A = \begin{bmatrix} 2 & 5 & 50 \\ 1 & 3 & 27 \\ 16 & 45 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 1 & 16 \\ 5 & 3 & 45 \\ 50 & 27 & 1 \end{bmatrix}$$

Example 3: Given the following matrices, find their transpose.

a. $B = \begin{pmatrix} -3 & 0 & 6 \\ 10 & 100 & 3 \end{pmatrix}$

b. D = $\begin{pmatrix} 0 \\ -4 \\ 11 \\ -3 \end{pmatrix}$

Equality of Matrices

Two matrices are equal if they have the same dimension and their corresponding entries are equal.

Example 4: Solve the following matrix equation for w, x, y, and z.

$$\begin{bmatrix} w+6 & x \\ y-2 & z \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}$$

Example 5: Solve for the variables in the matrix equation.

$$-\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} + 9\begin{bmatrix} u-6 & 2z+5 \\ y & -\frac{1}{3} \end{bmatrix} = -2\begin{bmatrix} 3 & -8 \\ 1 & v \end{bmatrix}$$